Nonlinear Hall effect in a time-periodic electric field and related phenomena.

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A motion of neutral and charged particles located nearby a metal surface under joint action of time-periodic electric field $E(t)$ directed normally to the surface and permanent magnetic field H directed along the surface has been considered. It has been shown that due to a nonlinearity of the system there exists a directed transport of the particles perpendicularly to both magnetic and electric fields with velocity which is proportional to E^2H. The velocity has been evaluated for charged (ions, electrons etc.) and neutral (atoms, molecules, nano-size clusters etc.) particles adsorbed on a metal surface. Corresponding surface electric current has been found as a function of a frequency of the electric field and material parameters. It has been noted that the similar phenomena appear in a bulk of electrically inhomogeneous media.

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The directed transport of particles under action of a time-periodic force is now well established phenomenon in various non-linear systems. The details of this transport has been studied for one-dimensional motion of particle in the space-periodic potential $U(x)$ and a time periodic force having general form $F(t) = F_0 \cos\omega t + F_1 \cos(2\omega t + \theta)$. (See review [1] and recent works [2–5].) Main physical reason for such directed motion of the particles is due to the so called latent asymmetry of driving force $F(t)$. Though the time average of $F(t)$ is zero the time average $\overline{F^3(t)}$ doesn't vanish and depends on θ. Nonlinearity of the system leads to mixing of harmonics such a way that average velocity is not zero and directed along $\overline{F^3(t)}$.

The aim of this paper is to show that the similar time- asymmetry arises in a permanent magnetic field even in the case of purely harmonic external force $F(t)$. The phenomenon is closely related with a conventional Hall effect in conducting media. In difference with the latter the DC electric current is arising on interface under a joint action of the harmonic electric field directed normally to the surface and a permanent magnetic field directed along the surface. The effect is of the second order with respect to the amplitude of the electric field. For this reason we'll call it the nonlinear Hall (NHL) effect.

Firstly we calculate the transverse velocity of a single charged particle located on a metallic surface in the AC electric field and longitudinal permanent magnetic field. Then, in Sections 2,3 we consider the similar phenomenon in a surface electron gas of metals and semiconductors. In Section 4 we evaluate the transport velocity of neutral particles in above conditions. Finally we give a phenomenological description of the NLH effect in inhomogeneous conducting media (Section 5).

I. DIRECTED MOTION OF SINGLE CHARGED PARTICLE.

In order to demonstrate the physical mechanism of DC current let us consider a motion of a single ion with a mass M on metallic surface. A complete description of an interaction between the ion and metallic surface is a quite complicated problem. In framework of a density functional approach it has been discussed in a recent monograph by Liebsch

[6]. There are a few solidly established facts we are going to use in our paper. The interaction between the ion and the metallic surface consists mainly of two parts. The first part is an attractive Coulomb force between the ion and an image charge having a potential $V_{im}(z)$

$$V_{im}(z) = -Q^2/(2z\epsilon) \tag{1}$$

where z is a distance between the ion and an edge of the metal surface and ϵ is a static dielectric constant. We gave here the simplest expression for such force at large z neglecting the dynamics of the metal electrons. More rigorous consideration includes an integration over the complex surface polarisability, which modifies the z- dependence of $V_{im}(z)$. For the sake of simplicity we will use (1) with some dimensionless parameter c_1

$$V_{im}(z) = -c_1 Q^2/z \tag{2}$$

The second part of the interaction is connected with a short-range repulsive exchange potential $V_{ex}(z)$ which we choose in exponential form

$$V_{ex}(z) = c_2 a_1 \exp\{-(z - z_e)/a_1\} \tag{3}$$

were z_e is an equilibrium distance between the physisorbed ion and metal surface, a_1 is a radius of exchange forces on surface. The c_2 is chosen from a equilibrium condition

$$V(z) = V_{im}(z) + V_{ex}(z) \tag{4}$$

$$\partial V/\partial z|_{z=z_e} = 0 \tag{5}$$

$$c_2 = c_1 Q^2/z_e^2 \tag{6}$$

The motion of the physisorbed ion with a velocity ν leads to electronic excitations in the metal and therefore to a friction force for such motion.

The friction force may be written as [6]

$$\mathbf{f} = -M(\eta_\parallel \nu_\parallel + \eta_\perp \nu_\perp) \tag{7}$$

Here ν is the particle velocity and $\eta_\parallel$, $\eta_\perp$ are the friction coefficients for the motion parallel and normal to the surface. If the charge is located outside the electronic density of metal, it can be shown that $\eta_\parallel = \eta_\perp/2$ [6]. At large ion surface distances z the friction parameter $\nu_\perp$ depends on z as

$$\nu_\perp = \frac{Q^2}{M} \frac{\omega_F}{k_F \omega_p^2 l} \frac{1}{z^3} \tag{8}$$

where $\hbar\omega_F, k_F, l$ and ω_p are a Fermi energy, a Fermi momentum, a mean free path of electrons and a plasmon frequency respectively.

For rough estimation it is convenient to use more simple formula

$$\nu_\perp = \frac{Q^2}{M \sigma_0 z^3} \tag{9}$$

Here σ_0 is a bulk conductivity of a metal. The latter gives a correct order of magnitude for $\nu_\parallel$ and $\nu_\perp$.

Having in mind all these facts one can readily write the equations of motion of the ion under a joint action of the AC electric field $E_0 \cos\omega t$ directed along z (normally to the surface) and a permanent magnetic field directed parallel to the surface along y-axis, $H_y = H$.

$$M\ddot{z} = -M\nu(z)\dot{z} + \frac{1}{c}Q\dot{x}H - \frac{\partial V}{\partial z} + \xi_z(t) + QE_0(t) \tag{10}$$

$$M\ddot{x} = -Mv_\parallel(z)\dot{x} + \frac{Q\dot{z}H}{c} + \xi_x(t) \tag{11}$$

Here c is a light velocity. We included also into these equations the random forces $\xi_{x(z)}(t)$ which reproduce the thermal fluctuations of the ion velocity. As usual, they have the white noise correlations of the following type

$$\begin{aligned} &< \xi_x(t_1)\xi_x(t_2) >= 2\nu_\parallel(z) k_B T \delta(t_1 - t_2) \\ &< \xi_z(t_1)\xi_z(t_2) >= 2\nu_\perp(z) k_B T \delta(t_1 - t_2) \end{aligned} \tag{12}$$

where k_B is a Boltzmann constant and T is a temperature of the system.

In spite of the comparative complexity of these equations they can be analyzed analytically in the limit of the very small electric and magnetic fields. At small amplitudes E_0, H of external fields a displacement of the ion $\delta z = z - z_e$ from its equilibrium position is quite small and the first equation (10) could be chosen in a linear form

$$M\ddot{z} = -2M\dot{z}\nu_0 - k_e\delta z + QE_0(t) \tag{13}$$

where k_e is a force constant of a vibration of ion along the z-axis

$$k_e = \frac{\partial^2 V}{\partial z^2}\Big|_{z=z_e} \tag{14}$$

and

$$\nu_0 = 2\nu_\perp(z_e) \tag{15}$$

We neglected in (10) the thermal fluctuations ($T = 0$) and the Lorentz force term. This is correct if we are interested in a linear contribution with respect to magnetic field into the velocity of the transport. However we have to keep this term in (11).

The nonlinearity of Eq.(11) is connected with damping term, which in accepted approximation could be written as

$$-M\nu_0(1-\gamma\delta z)\dot{x} \tag{16}$$

Here γ is a derivative of the damping function ν taken at equilibrium distance z_e. Then, the Eq.(11) took the following form

$$M\ddot{x} = -M\nu_0(1-\gamma\delta z)\dot{x} + \frac{QH}{c}\dot{z} \tag{17}$$

The periodic solution of (13) is

$$\delta z(t) = Re\Big\{\frac{E_0(Q/M)e^{i\omega t}}{\omega_e^2 - \omega^2 + i\nu_0 2\omega}\Big\} \tag{18}$$

Here ω_e is an eigen-frequency of the physisorbed ion, $\omega_e^2 = k_e/M$. Then the periodic solution of the second linear equation (17) could be written as the following

$$\begin{aligned}\dot{x} = \; & \int_0^\infty d\xi[QH/cM]\exp[-\xi\nu_0]\delta\dot{z}(t-\xi)\times \\ & \exp\{-\gamma\nu_0\int_{t-\xi}^t \delta z(\tau)d\tau\}\end{aligned} \tag{19}$$

We are interested in the transverse velocity $\dot{x}$ averaged over the period of the vibration in the external electric field $E_0(t)$. Making use the necessary averaging we have a final expression

$$\overline{v}(\omega) = \frac{Q^3E_0^2H\gamma}{2cM^3}\frac{\omega^2}{\omega^2+\nu_0^2}\frac{1}{(\omega_e^2-\omega^2)^2+4\omega^2\nu_0^2} \tag{20}$$

The function $\overline{v}(\omega)$ has a resonance character and reaches its maximal value at $\omega = \omega_e$. Since $\nu_0 \sim M^{-1}$ and $\omega_e^2 \sim M^{-1}$ the $\overline{v}(\omega_e)$ depends quite weakly on the mass of ion. As a rule, $\nu_0 \ll \omega_e$ (damping is comparatively small) and

$$\overline{v}(\omega_e) = \frac{Q^3E_0^2H\gamma}{2cM^3 4\omega_e^2\nu_0^2} \tag{21}$$

The behavior of $\overline{v}(\omega)$ as function of ω is given in Fig.1 for three different values of mass. On the same figure we gave the results of numerical solutions of Eqs.(10,11).

Let us estimate the possible velocities $\overline{v}$ at reasonable experimental parameters

$$M \sim 10^{-24}g, \qquad \gamma \sim 10^8 cm^{-1},$$
$$\omega_e \sim 10^{12}sec^{-1}, \qquad \nu_0 \sim 0.01\omega_e$$

and Q is an electron charge. Then

$$\overline{v}(\omega_e) \sim 10^{-3}E_0^2H,$$

where E_0 and H have to be taken in emu system. For example, if $H \sim 10^3 Gauss$ and $E_0 \sim 100V/cm$, $\overline{v}(\omega_e) \sim 1$ cm/sec.

In order to clarify the physical source of the directed transport of the particle we give a qualitative analysis of the equations(13) and (17). First of all we neglect the dependence of the damping coefficient $\nu(z)$ on z, $\gamma = 0$. Then, system of equations (13) and (17) became a linear one. The solution of this system can be written as following

$$\begin{aligned}\delta z &= a_0(\omega)\cos(\omega t + \varphi_z(\omega)) \\ x(t) &= a_0(\omega)b_0(\omega)\cos(\omega t + \varphi_z(\omega) + \varphi_x(\omega))\end{aligned} \tag{22}$$

where $a_0(\omega)$ and $\varphi_z(\omega)$ are modulus and argument of a complex function

$$\frac{E_0 Q}{M}\frac{1}{\omega_e^2-\omega^2+i2\nu_0\omega}$$

and $b_0(\omega)$ and $\varphi_x(\omega)$ are modulus and argument of a complex function $\dfrac{\omega_L}{\nu_0+i\omega}$.

A closed trajectory of the particle in the plane $(\delta z, x)$ is an ellipse obeyed the following equation (Fig.2)

$$\frac{(x-\delta z b_0\cos\varphi_x)^2}{a_0^2 b_0^2\sin^2\varphi_x}+\frac{\delta z^2}{a_0^2}=1 \qquad (23)$$

If either ω_L/ν_0 or ω/ν_0 trends to zero the ellipse turns into a line. An angle ψ between axes z and a largest axis of the ellipse is proportional to $b_0(\omega)$. Points A and A' on the Fig.2 are the points of return of the trajectory in the x-direction. At points C and C' the trajectiry cross the line $\delta z=0$, and dashed lines AB and $A'B'$ are parallel to this line. In fact, the ellipse (23) is an attractor of the system (13),(17).

Consider a small deformation of the trajectory taking into account the dependence of $\eta(z)$ on z. According to (16)the damping coefficient $\eta(z)$ is higher at $\delta z<0$ $(\gamma>0)$ and lower at $\delta z>0$. After switching on this dependence the trajectory of the particle between points C and B will be shifted a little bit to the right side. But this shift will be compensated completely by the shift to the left side on AC' part of the trajectory. The same is true for AC and $C'B'$ parts of the trajectory. Non-zero shift originates from AB and $A'B'$ parts. Indeed the velocity on the AB part is positive and the trajectory will be shifted to the right side if we diminish the damping coefficient of the particle. The velocity on $A'B'$ is negative and if we increase the damping the trajectory will be shifted also to the right side. All in all it leads to the shift of whole trajectory to the right which is equivalent to the transport of particle in this direction. The qualitative consideration given above does not depends on details of the attractor and holds for any non-linear dynamics of the system at.

Averaging the Eq.(17) over period of time we arrive to the following equality for the shift in x-direction

$$\overline{\delta x}=\gamma\int\delta z\dot{x}dt \qquad (24)$$

The integral in (24) is nothing but an area of the ellipse S and we have a very simple formula

$$\overline{\delta x}=\gamma S \qquad (25)$$

Eq.(25) could be used directly for the calculation of $\overline{v}(\omega)$ and of course it leads to the same result as (20).

Two additional remarks to the main expression (20). If the external AC electric field is a sum of two electric fields with two different amplitudes and frequencies ω_1 and ω_2 the effect is additive

$$\overline{v}=\overline{v}_1(\omega_1)+\overline{v}_2(\omega_2)$$

where $v_{1(2)}(\omega_{1(2)})$ are average velocities (20) with $E_0=E_{1(2)}$ and $\omega=\omega_{1(2)}$, correspondingly. Moreover, any non-thermal electric noise in system leads to a directed transport of charged particles. Corresponding average velocity could be obtained by integration of (20) over a frequency distribution of the electric noise.

Now, if we have two weakly coupled particles with opposite charges and different masses M_1 and M_2 the average velocity of their mass centre can be defined as

$$\overline{v}(\omega)=\frac{M_1\overline{v_1}(\omega)-M_2\overline{v_2}(\omega)}{M_1+M_2}$$

leading to some neutral current in system. More realistic situation with strongly coupled charged particles will be considered in Section 4.

II. THE NLH EFFECT ON METAL SURFACE.

The similar phenomenon takes play inside a metal on the boundary separating two media hav-

ing different conductivities. In this section we consider the metal-insulator or metal-vacuum boundary as simplest example of such interface. Consider dynamics of electronic gas in surface layer of metal under an influence of external AC electric field $E_0(t) = E_0 \cos \omega t$ directed perpendicularly to surface of the metal (along z axis) and in the magnetic field H directed along the surface, axis y. At this geometry a density of electronic gas, $\rho(z,t)$, is a function of normal coordinate z and a time t. There are two velocity components of the electronic gas: the normal component $v(z,t)$ and the tangential component $u(z,t)$ both of which are functions of only z and t. The electronic gas vibrates perpendicularly to the surface. Due to the nonlinearity of this vibrations a normal velocity $v(z,t)$ and density of electron gas $\rho(z,t)$ are periodic functions of time with frequency ω having all overtones:

$$v = v_1(z) \cos(\omega t + \varphi_1) + v_2(z) \cos(2\omega t + \varphi_2) \ldots$$

$$\rho = \rho_0 + \rho_1(z) \cos(\omega t + \varphi_3) + \rho_2(z) \cos(2\omega t + \varphi_4) \ldots$$

The above amplitudes and phases of harmonics as functions of z are a subject of calculations. The Lorentz forces induced by magnetic field lead to tangential vibration of electronic gas with amplitude which is proportional to permanent magnetic field. Corresponding tangential velocity $u(z,t)$ has the same form as $v(z,t)$ with its own amplitudes and phases. The density of tangential current is $j(z,t) = u(z,t)\rho(z,t)$. The time averaging of $j(z,t)$ over period gives the DC component of the current.

Since the width of a surface layer and all other characteristic lengths greater than atomic distances one can readily use hydrodynamics approach (see [6,7]) for description of system. In planar geometry all variables are the functions of only one coordinate z and a periodic functions of t with frequency ω. The main equations look as following [6,7]

$$\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial \rho}\frac{\partial \rho}{\partial z} + \frac{2\pi e}{m}Q(z)$$
$$-\frac{e}{m\mu}\rho v - \frac{e}{m}E_0(t)\rho - \frac{He}{mc}\rho u + \eta\frac{\partial^2 v}{\partial z^2} \qquad (26)$$

$$\rho\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial z}\right) = \frac{He}{mc}\rho v - \frac{e}{m\mu}\rho u + \eta\frac{\partial^2 u}{\partial z^2} \qquad (27)$$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial z}(v\rho) \qquad (28)$$

$$Q(z,t) = 2q(z) - q(\infty) \qquad (29)$$

Here p is a pressure of electronic gas in metal, μ is a mobility of electron in metal, c is the light velocity, m is effective mass of electrons, η is a viscosity coefficient of an electron fluid. A quantity

$$q(z,t) = e\int_0^z (\rho(z',t) - \rho_0)dz' \qquad (30)$$

is a surface density of charge (together with positive charge of a lattice) accumulated in a layer between $z = 0$ and z. The boundary conditions for this system are quite simple. Both velocities v, u are equal to zero at the edge of metal ($z = 0$).

We are looking for a periodic solution of the equations (22-25). The surface electric current directed along the surface and averaged over the period of time is

$$I_H = \int_0^\infty e\overline{\rho(z,t)u(z,t)}dz \qquad (31)$$

The bar in Eq.(31) means the time averaging.

The equation (28) can be rewritten via the $q(z,t)$ as

$$\frac{dq}{dt} = -ev(z,t)\rho(z,t) \qquad (32)$$

From this equation we see immediately that the time averaged normal component of the current

$$\overline{j}_z(z) = 0$$

Then, using the Eq.(27) we can get the following formula for time averaged tangential component of the current

$$\omega_\mu \overline{j}_x(z) = -e\frac{\partial}{\partial z}\overline{[\rho(z,t)u(z,t)v(z)]} + e\eta\overline{\frac{\partial^2 u}{\partial z^2}} \quad (33)$$

The full surface electric current I_H can be obtained by integration over z and looks as following

$$I_H = -\frac{e\rho_0}{\omega_\mu}\overline{u_\infty v_\infty} - \frac{e}{m\omega_\mu}\frac{\partial\eta}{\partial\rho}\int_0^\infty \overline{\frac{\partial\rho}{\partial z}\frac{\partial u}{\partial z}}dz \quad (34)$$

where $v_\infty(t)$ and $u_\infty(t)$ are the normal and tangential velocities for large z. The constant

$$\kappa = m/\frac{\partial\eta}{\partial\rho}$$

has a dimension of a kinematic viscosity cm^2/sec. At small electric and magnetic fields first non-vanishing contribution to the I_H can be obtained if we know $q(z,t)$, $u(z,t)$ and $v(z,t)$ in a linear approximation.

Taking the limit $z \to \infty$ we come to three equations for asymptotic values $q_\infty = q(\infty,t)$, $u_\infty = u(\infty,t)$, $v_\infty = v(\infty,t)$

$$\frac{d}{dt}q_\infty = -\rho_0 v_\imath \quad (35)$$

$$\frac{d}{dt}v_\infty = -v_\imath\omega_\mu + \frac{e}{m}E_0(t) - 2\beta q_\imath(t) \quad (36)$$

$$\frac{d}{dt}u_\infty = v_\imath\omega_L - u_\imath\omega_\mu \quad (37)$$

Here $\omega_\mu = e/m\mu$, $\beta = 2\pi e^2/m$ and $\omega_L = eH/mc$ is a Larmor frequency. Periodic solutions of these linear equations give the boundary conditions at $z \to \infty$. And $q(0,t) = 0$ by definition. Since one needs only the periodic solution of (26-29) the initial conditions are arbitrary. The system of equations (26-29) has been analyzed both by perturbation theory with respect to external electric field and numerically using standard NAG routines.

The periodic solution of linear equations (35-37) can be written in following form

$$\begin{aligned} v_\infty(t) &= c_e Re[a(\omega)e^{i\omega t}] \\ q_\infty(t) &= [\epsilon_0 c_e/\omega_\mu]\rho_0 Re[b(\omega)e^{i\omega t}] \\ \eta_\infty(t) &= \epsilon_0 h c_e Re[c(\omega)e^{i\omega t}] \end{aligned} \quad (38)$$

Here we introduced a dimensionless functions

$$\begin{aligned} a(\omega) &= \frac{\omega_\mu\omega\epsilon_0}{(\omega_s^2-\omega^2)+i\omega\omega_\mu} \\ b(\omega) &= \frac{i\epsilon_0\omega_\mu^2}{(\omega_s^2-\omega^2)+i\omega\omega_\mu} \\ c(\omega) &= a(\omega)\frac{\omega_L}{\omega_\mu+i\omega}, \end{aligned} \quad (39)$$

where ϵ_0 and h are the dimensionless electric and magnetic fields

$$\epsilon_0 = \frac{eE_0}{\omega_\mu c_e m}, \quad h = \frac{eH}{mc\omega_\mu}, \quad \omega_s^2 = \frac{2\pi e^2\rho_0}{m}$$

Using (26-29) as boundary conditions at an infinite z we arrive to the following solutions of the Eqs.(26-29)

$$\begin{aligned} v(z,t) &= c_e Re[a(\omega)e^{i\omega t}(1-e^{-\xi k})] \\ q(z,t) &= \rho_0 Re[b(\omega)e^{i\omega t}(1-e^{-\xi k})] \\ u(z,t) &= c_e Re[c(\omega)e^{i\omega t}(1-e^{-\xi k})] \end{aligned} \quad (40)$$

Here ξ is a dimensionless coordinate, $z = c_e\xi/\omega_\mu$, and complex $k = k_1 + ik_2$ is defined from the equation

$$(k_1+ik_2)^2 = [2\omega_s^2 - \omega^2 + i\omega\omega_\mu]/\omega_\mu^2, \quad k_2 > 0, \quad (41)$$

We are choosing a root which has $k_2 > 0$. Omitting intermediate calculations we give a final expression for I_H in following form

$$I_H = -\frac{ec_e^2\rho_0}{2\omega_\mu}h{\epsilon_0}^2\gamma_1(\omega) + \frac{\kappa\rho_0 e}{2}\gamma_2(\omega); \quad (42)$$

γ_1 and γ_2 are dimensionless functions of a frequency ω of the external field

$$\begin{aligned} \gamma_1(\omega) &= a^*(\omega)c(\omega) + c^*(\omega)a(\omega); \\ \gamma_2(\omega) &= [k^2k^*c^*(\omega)b(\omega) + h.c.][k+k^*]^{-1} \end{aligned}$$

Both of them have resonance character and reach their peaks at $\omega = \omega_s$ as it shown on Fig.3. Maximal value of them are $\gamma_1(\omega_s) = 2\omega_\mu^2/\omega_s^2$ and $\gamma_2(\omega_s) = 1$. For metals at low temperature as a rule $\omega_\mu^2/\omega_s^2 \ll 1$. At very high frequency $\omega \gg \omega_s$

both $\gamma_1(\omega)$ and $\gamma_2(\omega)$ decrease as $1/\omega^4$. At low frequency $\omega \ll \omega_\mu$ they behave as ω^2: $\gamma_1(\omega) \sim \omega_\mu^2\omega^2/\omega_s^4$ and $\gamma_2(\omega) = \omega^2/\omega_s^2$.

Thus, at low temperature main contribution to the surface current I_H comes from the second term of (38), depending on the viscosity of electronic gas. According to [8] the viscosity of electronic Fermi liquid in metal at low temperature could be estimated as

$$\eta \cong m\rho_0 c_e d(\epsilon_F/T)^2 \tag{43}$$

where d is a lattice constant and ϵ_F is a Fermi energy of a metal.

Thus, at low temperature the surface current I_H can be expressed as

$$I_H = -\frac{\omega^2 e\rho_0 dc_e}{4\omega_s^2}\left(\frac{\epsilon_F}{k_B T}\right)^2\left(\frac{\mu E}{c_e}\right)^2\left(\frac{\mu M}{c}\right)$$

It is easy to see that the temperature dependence of the effect is determined by the factor $\sigma^3(T)/T^2$ ($\sigma(T)$ is a conductivity of the metal).

To demonstrate the value of the effect let us take material parameters of pure copper at low temperature, say, T=30 K:

$$d = 10^{-8}\ cm, \quad c_e = 10^8\ \frac{cm}{sec}, \quad \rho_0 = 10^{22}\ cm^{-3}.$$

The conductivity behave as T^{-5}:

$$\sigma(T) = \sigma_R(T_R/T)^5$$

where $\sigma_R = 10^6\ om^{-1}$ is a room temperature conductivity and $T_R = 300\ K$.

Finally, for the pure cooper at $T = 300\ K$ we have

$$I_H = -0.1\nu^2 E_0^2 H$$

where ν is expressed in MHz, E_0 in a V/cm, H in a $Tesla$, I_H is in a $\mu A/cm$. At $E_0 = 10\ V/cm$, $H = 0.01\ T$, $\nu = 10\ MHz$ one has $I_H =\sim 1.0\ \mu A/cm$.

III. NLH CURRENT IN AN INJECTION LAYER OF A SEMICONDUCTOR.

In the previous Section we have considered the dynamics of electron gas inside a metal on the boundary metal-semiconductor. If a work function for transition of electrons from metal side to the semiconductor is not too high electrons of the metal can be injected into the semiconductor creating an injection layer of electrons inside of the semiconductor. The injection layer can give a contribution to the NLH current on the interface metal-semiconductor. An estimation of the NLH current in this injection layer is the subject of present section. First of all, let us consider the equilibrium electron density in the semiconductor side. Free energy of electron gas per unit area of the surface has the following form [6, Liebsch]

$$F = \int_0^\infty f(\rho,T)dz + 2\pi e^2\int_0^\infty \rho(z)dz\int_0^\infty z\rho(z)dz - e^2\pi\int_0^\infty\int_0^\infty \rho(z)|z-z'|\rho(z')dzdz' + \psi_0\int_0^\infty \rho(z)dz \tag{44}$$

where $f = (\rho(z),T)$ is a density of the free energy of injected electron gas, which is a function of electron density $\rho(z)$ and temperature T. At small density $\rho(z)$ the $f = (\rho,T)$ is a free energy of ideal classical gas.

$$f(\rho,T) = T\rho\ln\rho/\rho_T$$

We use the units where Boltzmann constant $k_B = 1$. Ψ_0 is.a work function for transfer electron from metal to the semiconductor. Minimization of F with respect to $\rho(z)$ leads to the well known equation for equilibrium density $\rho_e(z)$

$$\frac{\partial f}{\partial\rho} + 2\pi e^2 z\int_0^\infty \rho_e(z)dz - e^2 2\pi\int_0^\infty \rho_e(z')|z-z'|dz' + 2\pi e^2\int_0^\infty z\rho_e(z)dz + \Psi_0 = 0 \tag{45}$$

For Boltzmann electron gas the solution of (45) looks as following

$$\rho_e(z) = \frac{z_d q_\infty^2}{(z z_d q_\infty + 1)^2} \tag{46}$$

Here $q_\infty = \int_0^\infty \rho(z) dz$ and

$$z_d = \frac{2\pi e^2}{T},$$

is full injected charge per unit of area of surface. An equality of chemical potential of the metal and the semiconductor on the surface of the metal

$$\left.\frac{\partial f}{\partial \rho}\right|_{\rho=\rho_e(0)} = -\Psi_0 \tag{47}$$

gives

$$\rho_e(0) = \rho_T e^{-\Psi/T} \tag{48}$$

and

$$q_\infty = (\rho_T/z_d)^{1/2} e^{-\Psi/2T} \tag{49}$$

In fact, the equations (45-49) summarise briefly the simplest theory of Schottky barrier on the metal-semiconductor interface [9]. Under action of time-periodic external electric field $E_0 \sin\omega t$ the non-linear oscillations of the density $\rho(z,t)$ takes place. A system of equations describing these oscillations looks very much like that of the metal-vacuum interface equations (26-29), but the background positive charge ρ_0 in Eq.(29) has to be taken zero.

Another important difference between the metal-insulator and metal-semiconductor cases is connected with boundary conditions for velocities $v(z,t)$ and $u(z,t)$ at $z = 0$. There are two limit cases. In the first one, the exchange of charges over the surface is quite slow which means that total charge in the semiconductor is conserving under action of the AC electric field and $v(0,t) = 0$, $u(0,t) = 0$ at the boundary. If we neglect the viscosity a total NLH current vanishes.

Let us outline briefly the solution of the hydrodynamics equations (26-29) in this case. At small electric E_0 and magnetic H fields and small viscosity of electron liquid the first equation (26) can be rewritten as

$$\rho\omega_\mu = -c_e^2 \frac{\partial \rho}{\partial z} + \rho 4\pi(\rho(z) - \rho_\infty) + eE(t)\rho \tag{50}$$

where

$$\rho(z,t) = \int_0^\infty \rho(z) dz \tag{51}$$

and

$$c_e^2 = \rho \frac{\partial^2 f}{\partial \rho^2} \tag{52}$$

For Boltzmann gas $c_e^2 = \sqrt{T/m}$. The equation of continuity can be rewritten as

$$\frac{\partial q}{\partial t} = -v\rho \tag{53}$$

In the equation for $u(z,t)$ we have to keep small terms connected with the magnetic field (Lorentz force) and viscosity. Then, this equation can be rewritten in the following form

$$\frac{\partial u\rho}{\partial t} + \frac{\partial}{\partial z} vu\rho = \omega_L v\rho - \omega\mu u\rho + \frac{\eta}{m}\frac{\partial^2 u}{\partial z^2} \tag{54}$$

Having in mind that we are interested in the full surface current averaged over period of oscillation of external electric field

$$I_H = e\int_0^\infty \overline{\rho u} dz \tag{55}$$

we arrive to the expression

$$I_H = \frac{e}{\omega_\mu}\int_0^\infty \frac{\eta}{m}\frac{\partial^2 u}{\partial z^2} dz \tag{56}$$

Here again the bar means the averaging over period of oscillation. We took into account also that

$$\overline{j_z} = \overline{v\rho} = 0 \tag{57}$$

due to the second equation.

As a rule, the viscosity of electronic gas η depends on the density of electron and, thus, depend on z . It is more convenient to rewrite (56) as

$$I_H = -\frac{e}{\omega_\mu\, m}\int_0^\infty (\frac{\partial \eta}{\partial \rho})\frac{\partial \rho}{\partial z}\frac{\partial u}{\partial z} dz \tag{58}$$

Further we shall show that

$$\left.\frac{\partial u}{\partial z}\right|_{z=0} = 0$$

At small viscosity and small ω/ω_μ (low frequency of external field)

$$u = \frac{\omega_L}{\omega_\mu} v$$

and we can express the I_H via normal velocity $v(z,t)$ and density $\rho(z,t)$ found from equation (50)

$$I_H = -\frac{e\omega_L}{m{\omega_\mu}^2}\int_0^\infty (\frac{\partial\eta}{\partial\rho})\frac{\partial\rho}{\partial z}\frac{\partial u}{\partial z}dz \qquad (59)$$

The viscosity of electron gas is unknown function of density of the gas. Father, for estimation, we assume that is $\frac{1}{m}\frac{\partial\eta}{\partial\rho}$ is approximately equal to kinematic viscosity of electronic gas,

$$\kappa = \frac{1}{m}\frac{\partial\eta}{\partial\rho},$$

taken at equilibrium surface density of electron gas, $\rho_{eq}(0)$.Thus (60) turns into the following expression

$$I_H = -\frac{e\omega_L}{{\omega_\mu}^2}\kappa\int_0^\infty dz\frac{\partial\rho}{\partial z}\frac{\partial v}{\partial z} \qquad (60)$$

which we will use for estimation of nonlinear Hall surface current.

The solution of the eqs. (50) will be performed by theory of perturbation with respect to a small amplitude of electric field E_0 . After introduction of dimensionless time $\tau = \omega t$, coordinate $z = \frac{c_e\xi}{\sqrt{\omega\omega_\mu}}$, electric field $\epsilon_0 = \frac{eE_0}{mc_e\sqrt{\omega\omega_\mu}}$ and new function $\lambda(z,t)$.

$$q_{(z,t)} = q_\infty(1 - \alpha\lambda(z,t)) \qquad (61)$$

the (50) turns into the following equation

$$\frac{\partial\lambda}{\partial\tau} = \frac{\partial^2\lambda}{\partial\xi^2} + 2\eta\frac{\partial\lambda}{\partial\xi} + \epsilon\frac{\partial\lambda}{\partial\xi}\cos\tau \qquad (62)$$

which has to be solved with boundary conditions

$$\lambda(\xi) \to 0 \ \text{ at } \ \xi \to \infty$$

$$\lambda(0) = \frac{1}{\alpha}$$

Here we use the following notation

$$\alpha = \frac{\sqrt{\omega\omega_\mu}c_e m_0}{2\pi q_\infty e^2}, \beta = \frac{c_e}{\sqrt{\omega\omega_\mu}}$$

$$v = -\beta\omega(\frac{\partial\lambda}{\partial\tau})/(\frac{\partial\lambda}{\partial\xi})$$

Note that we are looking for the periodic solution of (63). If we make a substitution

$$\lambda = -(\frac{\partial\varphi}{\partial\xi})/\varphi \qquad (63)$$

the nonlinear eq. (63) turns into linear one

$$\frac{\partial\varphi}{\partial\tau} = \frac{\partial^2\varphi}{\partial\xi^2} + \epsilon\cos\tau\frac{\partial\varphi}{\partial\xi} \qquad (64)$$

with the boundary conditions

$$\varphi(\xi) \to \infty \ \text{ at } \ \xi \to \infty$$

$$\left.\frac{\partial\varphi}{\partial\xi}\right|_{\xi=0} = \frac{1}{\alpha}\varphi(0)$$

The perturbative analysis of linear Eq. (65) is rather trivial and we give only the final result

$$\lambda(\xi,t) = \lambda_0(\xi) + \epsilon\lambda_1(\xi,t) \qquad (65)$$

$$\lambda_0 = \frac{1}{\xi+\alpha} \qquad (66)$$

$$\lambda_1 = \epsilon\left[\frac{1}{\xi+\alpha}\frac{\partial a(\xi)}{\partial\xi} - \frac{1}{(\xi+\alpha)^2}a(\xi)\right]e^{i\tau} \qquad (67)$$

where $a(\xi)$ has the form

$$a(\xi) = i\left[\frac{1}{1+k\alpha}e^{-k\xi} - 1\right], k = \frac{i+1}{\sqrt{2}} \qquad (68)$$

Omitting intermediate calculations we give the expression for I_H in the following form

$$I_H = -e\frac{b\omega_L}{{\omega_\mu}^2}\frac{q_\infty\alpha}{\beta}\epsilon^2\int_0^\infty\frac{1}{(\xi+\alpha)^4}v(\xi)d\xi \qquad (69)$$

where

$$v(\xi) = \overline{\left(\frac{\partial\eta_1}{\partial\tau}\frac{\partial\eta_1}{\partial\xi}\right)}/\left(\frac{\partial\eta_0}{\partial\xi}\right)^2$$

since is proportional to $\sqrt{\omega}$,the function $R(\alpha)$,

$$R(\alpha) = \int_0^\infty \frac{1}{(\xi+\alpha)^4} v(\xi) d\xi \tag{70}$$

is actually a function of frequency of external electric field ωṄumerical calculation shows that $R(0) = -0.21$. Collecting all multipliers we get the following expression for the I_H

$$I_H = -\frac{3H}{2\pi c} R(0) \omega \kappa \left(\frac{\mu E_0}{c_e} \right)^2 \tag{71}$$

As an example let us choose reasonable parameters

$$\rho_\infty = 10^{12} \frac{1}{cm^3}, \qquad \kappa = 10^3 \ \frac{cm^2}{sec},$$

$$\mu = 10^5 \ \frac{cm^2}{V \sec}, \qquad v = 10^7 \ \frac{cm}{sec}.$$

Then, I_H can be written as

$$I_H = 10^{-5} H \omega E_0^2$$

where I_H is expressed in a $\dfrac{\mu A}{cm}$, H in a Oe, E_0 in a V/cm and ω in a MHz.

IV. MAGNETOPHORESIS OF NEUTRAL PARTICLES ON THE SURFACE.

Consider now the neutral particle (atom, molecule or any nano-size particle) located on the metal surface under joint action of normal AC-electric and tangent permanent magnetic fields. As it has been shown in Section 1 the driving force which causes the directed transport of particle on the surface depends on a mass, charge and details of an interaction between particles and the surface of metal. If negative and positive charges coupled in a neutral pair as it takes place in atoms and molecules then the driving forces acting on positive and negative charges are directed on opposite sides, but their values, generally speaking, can be different. Thus, the resulting force acting on the particle is not necessary zero leading to the directed transport of the particles. As it is well-known the Lorentz forces acting on the neutral particle in a magnetic field H can be written as

$$\mathbf{F}_l = \frac{1}{c} \left[\frac{\partial \mathbf{D}}{\partial t} \times \mathbf{H} \right] \tag{72}$$

where $\mathbf{D}$ is a dipole moment of the system of the charged particles. On the other hand, the dipole moment $\mathbf{D}$ of the neutral particle in an external electric field $\mathbf{E}_0 \cos(\omega t)$ is an algebraic sum of a permanent dipole moment of the particle $\mathbf{D}_0$ and the dipole moment $\mathbf{D}_1(t)$ induced by an external time-periodic electric field $\mathbf{E}_0 \cos(\omega t)$. Thus, we have

$$\mathbf{D}(t) = \mathbf{D}_0 + \alpha(\omega) \mathbf{E}_0 \cos(\omega t) \tag{73}$$

where $\alpha(\omega)$ is a frequency depending polarisibility of the particle. As a rule, for neutral particle absorbed on a metal surface vector $\mathbf{D}_0$ is directed normally to the surface. The potential of interaction between the dipole particle and metallic surface $V(z)$, consists of two terms [6].

The Coulomb attraction potential

$$V_{id}(z) = -\frac{\mathbf{D}^2}{4z^3 \epsilon} \tag{74}$$

and a repulsive exchange potential $V_{ed}(z)$, which we again (see Section 1) choose in an exponential form

$$V_{ed}(z) = c_d a_d \exp\left(-\frac{z - z_d}{a_d} \right) \tag{75}$$

Here a_d is a radius of exchange forces on the metallic surface and z_d is an equilibrium distance between the particle and the surface, c_d is determined from an equilibrium condition

$$V(z) = V_{id}(z) + V_{ed}(z), \qquad \left. \frac{\partial V}{\partial z} \right|_{z=z_d} = 0 \tag{76}$$

As in the case of ion-metal system the friction coefficient, ν, for the dipole particle located on a metal surface depends on the distance z (see [6]). At large distances z, the $\nu(z)$ is proportional to $1/z^3$.

Similar to Section 1 we choose this dependence in a form

$$\nu(z) = \nu_0[1 - \gamma_d(z - z_d)] \tag{77}$$

where $\nu_0 = \nu(z_d)$ and $\gamma_d = - \left.\dfrac{\partial \nu(z)}{\partial z}\right|_{z=z_d}$.

At small deviation from the equilibrium position of the particle the equations of motion for its gravity center can be written as following

$$\begin{aligned} M\ddot{z} &= -M\nu(z)\dot{z} - \omega_d^2 M(z - z_d) - Q_d E_0(t) \\ M\ddot{x} &= -M\nu(z)\dot{x} + \frac{H}{c}\frac{\partial D}{\partial t} \end{aligned} \tag{78}$$

Here ω_d is an eigenfrequency of particle vibration on the surface and

$$Q_d = \frac{3D_0\alpha(\omega)}{z_d^4}$$

Using (73) we get

$$\frac{\partial D}{\partial t} = -\omega\alpha(\omega)E_0 \sin(\omega t) \tag{79}$$

The system of equations (78) is complete analog of that in the Section 1. Omitting the details of the calculation we give the expression for tangent velocity $\frac{dx}{dt}$ averaged over a period of oscillation of the external effective electric field

$$\bar{\dot{x}} = v(\omega) = \frac{\gamma}{4}\left[g_0(\omega)g_1^*(\omega) + g_1(\omega)g_0^*(\omega)\right] \tag{80}$$

where complex functions $g_0(\omega)$ and $g_1(\omega)$ have form

$$g_0(\omega) = \frac{Q_d E_0}{M(\omega_d^2 - \omega^2 + i\nu_0\omega)} \tag{81}$$

$$g_1(\omega) = \frac{E_0\alpha(\omega)Hi\omega}{Mc(i\omega + \nu_0)} \tag{82}$$

Final expression of the $v(\omega)$ has a form

$$v(\omega) = \gamma\frac{3D_0}{z_d^4 M^2 c}\frac{\alpha^2(\omega)E_0^2 H\omega^2}{\omega^2 + \nu_0^2}\frac{\omega_d^2 - \nu_0^2 - \omega^2}{\left(\omega_d^2 - \omega^2\right)^2 + \nu_0^2\omega^2} \tag{83}$$

At small ω the velocity $v(\omega)$ is decreasing as ω^2 in complete analogy with corresponding velocity of charged particle. However, in difference with the Section 1 $v(\omega)$ changes its sign at $\omega = \sqrt{\omega_d^2 + \nu_0^2}$. The general form of $v(\omega)$ as a function of frequency is presented on Fig.4.

If the dipole moment of the particle D_0 is proportional to its volume (as it takes place for ferroelectric medium) then $v(\omega)$ does not change much with the increase of the radius of the particle and its mass.

If the external electric field $E_0(t)$ is a sum of two harmonics with different frequencies ω_1 and ω_2 the resulting velocity is additive,

$$v(\omega) = v_1(\omega_1) + v_1(\omega_2) \tag{84}$$

For atomic size particles (atom or molecule) the maximal velocity could be estimated as

$$v = 10^{-12}E_0^2 H \tag{85}$$

where v is given in cm/sec, E_0 in V/cm and H in Oe.

V. NLH EFFECT PHENOMENOLOGY.

In previous Sections we have shown that the joint action of the permanent magnetic and AC-electric fields leads to the direct transport of particles located nearly surface. An important role in consideration plays gradient of the damping constant the system metal-particle. Generalizing all examples of the previous Section we can assume that the NHL effect takes place in a bulk of the material if there exists a non-zero gradient of conductivity tensor j_i could be chosen as

$$j_i = \lambda(\omega)\frac{\partial \sigma_{ij}}{\partial z_k}\dot{E}_k\dot{E}_l\epsilon_{jlm}H_m \tag{86}$$

where $\dot{E}_k$ is a time derivative of an external electric field and H is magnetic field. In a case of isotropic medium $\sigma_{ij} = \delta_{ij}\sigma$ the density of current is

$$\mathbf{j} = \lambda(\nabla\sigma\dot{\mathbf{E}})[\dot{\mathbf{E}} \times \mathbf{H}] \tag{87}$$

Here λ is a phenomenological coefficient depending on frequency and material parameters.

To demonstrate the physical consequences of (88) consider the cylindrical system shown on Fig.5. The time periodic voltage is applied between inner and external surface of the cylindrical sample. The permanent magnetic fields H is directed along main axes of the cylinder. We assume also that the conductivity of the metal decreases along the radius of the sample, $\frac{\partial \sigma}{\partial r} < 0$.

In the chosen cylindrical geometry there exist only angular component of the current density $j_\varphi(r)$ which according to (88) can be expressed as

$$j_\varphi(r) = \lambda(\omega)\frac{\partial \sigma}{\partial r}\dot{\mathbf{E}}^2 \mathbf{H} \tag{88}$$

Now, let us assume also, that the conductivity σ , ω and thickness of the sample are chosen such a way that the electric field $E(t)$ penetrates over the sample.Then the additional magnetic field $H_{ad}(r)$ induced by this current can be written as

$$H_i(r) = -\frac{4\pi}{c}\int_{r_1}^{r} \overline{j_\varphi}(r)dr = \frac{4\pi}{c}(r - r_1)\overline{j_\varphi} \tag{89}$$

where r_1 is an radius of the sample. There exist also diamagnetic contribution to the induced magnetic field which is proportional to diamagnetic susceptibility, χ ,of the material

$$H_d = \chi H \tag{90}$$

Summing up both terms we come to the conclusion that the full induced magnetic field is positive if

$$-\lambda\frac{\partial \sigma}{\partial r}\overline{\dot{\mathbf{E}}^2}\frac{4\pi}{c}l_d > |\chi| \tag{91}$$

Actually, the condition (91) is a condition of stability of the system to the generation of magnetic field in the sample under action of time periodic electric field $E(t)$.The details of phenomenon will be considered in the future.

The main obstacle for observation of the NLH effect in AC electric field seems to be connected with an absorption of ultrahigh frequency electromagnetic field into material. The time-periodic components of Hall current which is linear with respect to external electric field is greater than a permanent component of the NLH current and may lead to heating up of the sample. On the other hand, at low (or ultralow) temperature the absorption and warming up could be made small enough for such observation.

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Captions to Figures

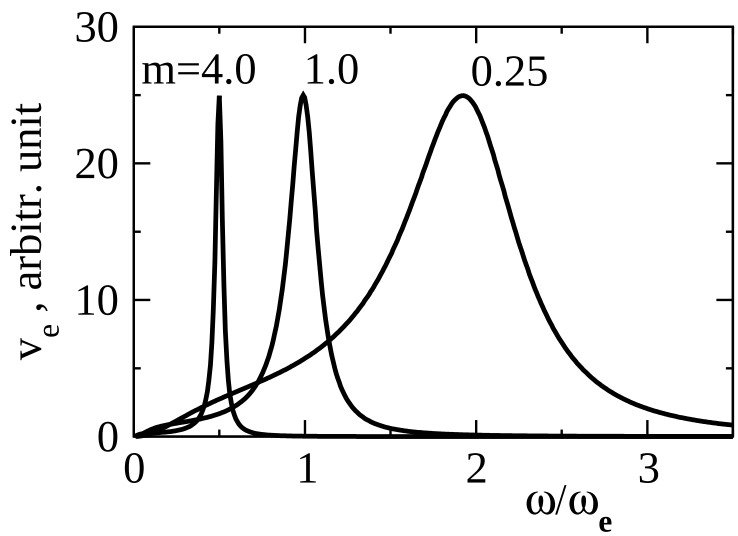
Fig. 1. Frequency dependence of the velocity (20) of a single charged particle at $\nu_0 = 0.1\omega_e$ for three different masses $m = M/M_0 = 0.25$, $M/M_0 = 1.0$ and $M/M_0 = 4.0$.

Fig. 2. An attractor of the system (13-17). The A and A' are points of return in x direction. Arrows on the trajectory show the direction of the particle rotation, ψ is an angle of inclination of the ellipse.

Fig. 3. Frequency dependence of $\gamma_1(\omega)$ and $\gamma_2(\omega)$ at $\omega_\mu = 0.1\omega_s$.

Fig. 4. Frequency dependence of the velocity (83) of neutral particle at $\nu_0 = 0.1\omega_d$.

Fig. 5. The principle set up for observation of NHL effect. Electric field $E_0(t)$ is directed along the radius of the cylinder sample.



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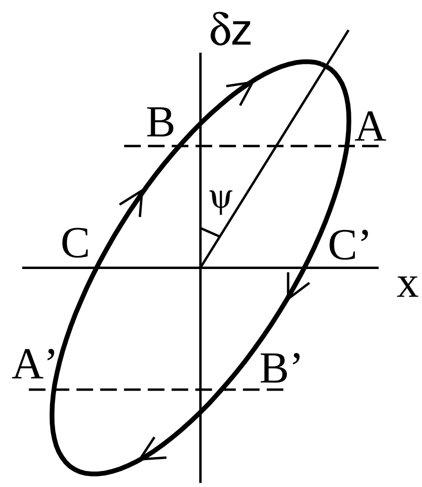

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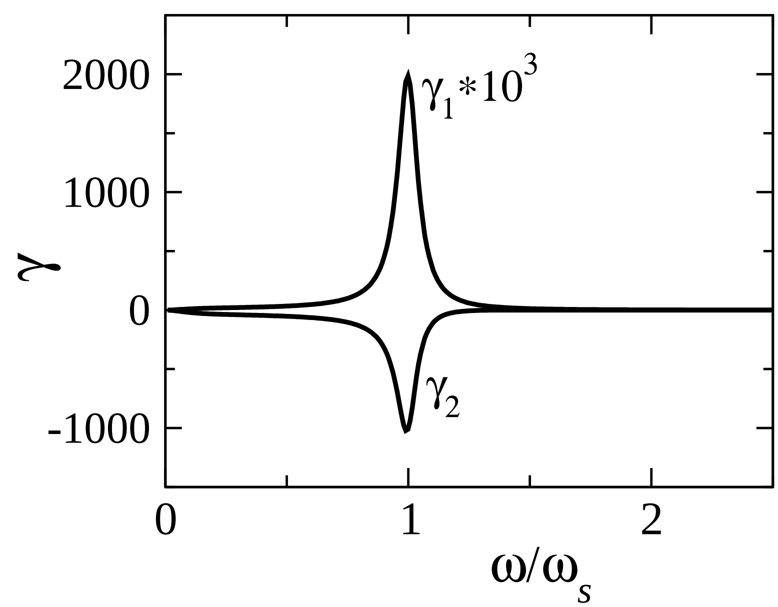

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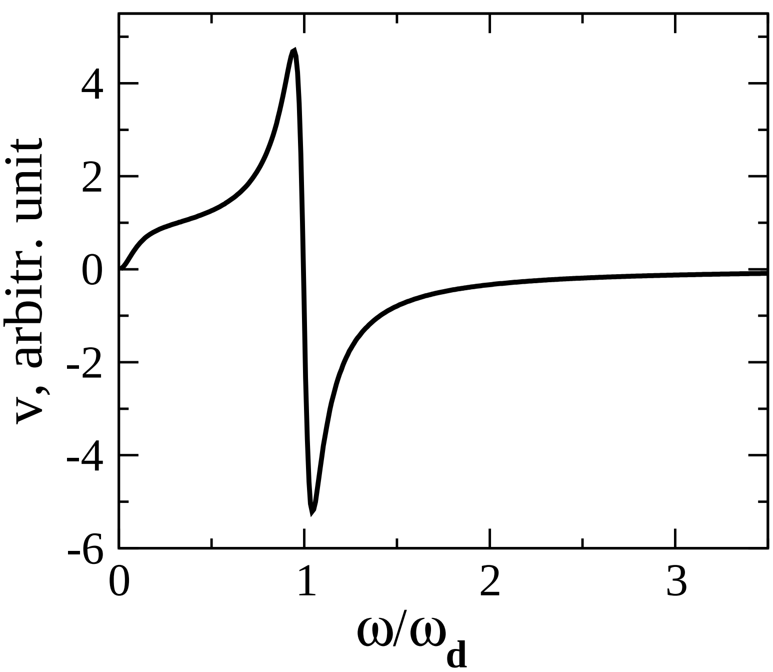

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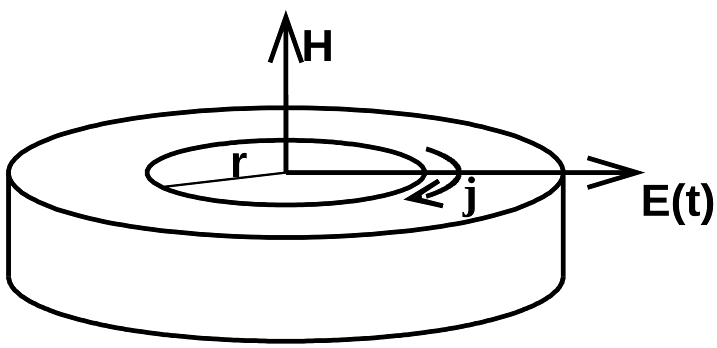
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